

MATH1520AB 2021-22 Quiz 6 (week 10)

Full marks: 10 marks

Time allowed: 15 minutes

1. Evaluate each of the following indefinite integrals.

(a) $\int (x^2 - \frac{1}{x})^2 dx$

(b) $\int (e^{3x} + 3x^e) dx$

Answer.

(a) $\int (x^2 - \frac{1}{x})^2 dx = \int (x^4 - 2x + x^{-2}) dx = \frac{x^5}{5} - x^2 - \frac{1}{x} + C$

(b) $\int (e^{3x} + 3x^e) dx = \frac{e^{3x}}{3} + \frac{3x^{e+1}}{e+1} + C$

2. Evaluate the following indefinite integrals using substitution.

(a) $\int x e^{x^2+1} dx$

(b) $\int \frac{x^2 - 1}{\sqrt{x^3 - 3x + 1}} dx$

Answer.

(a) Let $u = x^2 + 1$. Then $du = 2x dx$ and $\frac{1}{2} du = x dx$.

$$\int x e^{x^2+1} dx = \int \frac{1}{2} e^u du = \frac{1}{2} e^u + C = \frac{1}{2} e^{x^2+1} + C$$

(b) Let $u = x^3 - 3x + 1$. Then $du = (3x^2 - 3) dx$ and $\frac{1}{3} du = (x^2 - 1) dx$.

$$\int \frac{x^2 - 1}{\sqrt{x^3 - 3x + 1}} dx = \int \frac{du}{3\sqrt{u}} = \frac{1}{3} \left(\frac{u^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} \right) + C = \frac{2}{3} \sqrt{u} + C = \frac{2}{3} \sqrt{x^3 - 3x + 1} + C$$

3. Evaluate $\int x \ln x dx$ using integration by parts.

Answer.

$$\begin{aligned} \int x \ln x dx &= \int \ln x d\left(\frac{x^2}{2}\right) \\ &= \frac{x^2}{2} \ln x - \int \frac{x^2}{2} d(\ln x) \\ &= \frac{x^2}{2} \ln x - \int \frac{x^2}{2} \frac{dx}{x} \\ &= \frac{x^2}{2} \ln x - \int \frac{x}{2} dx \\ &= \frac{x^2}{2} \ln x - \frac{x^2}{4} + C \end{aligned}$$